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# Fraction-exponential representation of the viscoelastic properties of dentin

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## Abstract

We propose the fraction-exponential description of the viscoelastic properties of dentin. Creep tests are performed on specimens cut from the molar coronal part. Four parameters determining instantaneous and long term Young's moduli as well as the relaxation time are extracted from the experimental data. The same procedure is repeated using the experimental measurements of Jantarat et al (2002) for the specimens cut from the root part of incisor. Physical meaning of the parameters and the difference between them for different sets of specimens are discussed.

### Keywords

dentin; viscoelasticity; fraction-exponential operator; molar; incisor

## 1. Introduction

In the present paper we develop a fraction-exponential model for viscoelastic properties of dentin. Dentin represents the main part of tooth mineralized tissue with a rather complex hierarchical microstructure. The latter can change due to various diseases like caries or sclerosis, or/and due to aging and thus lead to changes in the mechanical performance of dentin. As mentioned by Kinney et al (2003), knowledge of dentin properties is also

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important for understanding the effects of the wide variety of restorative dental procedures in clinical dentistry: properties of restorative materials should be similar to those of the living tissue.

Chemically, dentin consists of approximately 70 wt. % inorganic material, 18 wt. % organic matrix and 12 wt. % water (Halgas et al 2013). Mechanical properties of dentin, as of any other material, are governed by its microstructure which has multiscale hierarchical character. It is characterized by the presence of tubules (~1.5 μm in diameter) that run from the dentin-enamel junction towards the pulp. The tubules are surrounded by highly mineralized cylinders of peritubular dentin, roughly 0.5-1 μm in thickness, composed largely of apatite. These tubules are separated by intertubular dentin that consists of a hydrated matrix of type I collagen which is reinforced with a nanocrystallites of carbonated apatite (Brauer et al 2011). The structural and compositional dissimilarities between the enamel and dentin induce significant differences in their mechanical behavior (Shahmoradi et al 2014). Quantitative understanding of the relationship between microstructure and mechanical properties of human dentin allows identification of the microstructural parameters governing the properties and leads to new methodologies in development of tissue equivalent materials.

To the best of our knowledge, the first systematic experimental study of mechanical properties of dentin under compression was performed by Black (1895) who, in particular, showed that location and orientation of the tubules of the test specimens do not significantly affect the overall properties. This result was later argued in works of Peyton et al (1952) and Stanford et al (1958). In the latter work, it was also shown that the dentin demonstrates anisotropic (transversely-isotropic) properties. Viidik (1968) gave a review of the mechanical properties of collagenous tissue (including dentin) and their relation to morphology. Balooch (1998) used an atomic force microscope to measure the mechanical properties of demineralized human dentin under three conditions: in water, in air after desiccation, and in water after rehydration. The experiments showed that contribution of collagen fibers into elastic stiffness of dentin is negligible, although collagen is a significant contributor to dentin strength and toughness. Bo et al (2000) studied elastic behavior of dentin and reported results of tensile experiments on the small dentin specimens either parallel or perpendicular to the dentin tubules. The determined effective Young's modulus and Poisson's ratio of dentin matrix were 29.5GPa and 0.44, respectively. Kinney et al (2003) provided a detailed review of the experimental results on the mechanical properties of human dentin obtained in the second half of XX century. The authors briefly discussed the composition and microstructure of dentin and then summarized results on its elastic properties, hardness, strength, fracture toughness, and fatigue. Viscoelastic properties were also discussed but not many results were mentioned.

Jantarat et al (2002) studied the creep, stress relaxation and strain rate behavior of human root dentin and showed that the viscoelastic behavior is linear. Pashley et al (2003) analyzed stress–relaxation curves in tension and came to the opposite conclusion – the dentin matrix exhibits viscoelastic properties, but it is not linearly viscoelastic. Duncanson and Korostoff (1975) used stress relaxation measurements to show that the relaxation modulus of dentin exhibits a linear dependence on the logarithm of time during a period of about four decades and that the distribution of relaxation times is constant to a high degree during this interval for the orientation investigated. Cui et al (2010) suggested to use a poro-viscoelastic mechanical model to describe dentine with no cracks. The model is able to predict creep strain, stress relaxation and instant elastic response with anisotropic constitutive relation for porous cylindrical composite materials. Zaitsev et al (2012) provided in vitro experimental results on elastic behavior of human dentin under compression including shape, size, and strain rate effects. Halgaš et al (2013) characterized the hardness, elastic modulus, the load size effect on hardness, load rate effect on deformation and indentation creep of human enamel and dentin using instrumented indentation methods. They showed that the indentation load rate had only a minor influence on the penetration depth/energy loss of enamel. The creep behavior of enamel at applied loads of 10, 50, 100 and 400 mN exhibits a relatively short primary creep region and a pronounced secondary region with a stress exponent of $n = 1.8$.

Recently, Singh et al (2015) and Singh (Master thesis, 2009) developed a linear viscoelastic model for collagen–adhesive composite and dentin adhesives and demonstrated the applicability of the model by predicting stress relaxation behavior, frequency-dependent storage and loss moduli, and rate-dependent elastic modulus. Jafarzadeh (2015) experimentally determined the viscoelastic characteristics of human dentin under the action of a uniaxial static compressive stress and showed that dentin exhibited a linear viscoelastic response under ‘clinical’ compressive stress levels with a maximum strain ~ 1% and high recoverability: permanent set<0.3%. Chuang et al (2015) proposed a quantitative approach to characterize the viscoelastic properties of dentin after demineralization, and to examine the elastic properties using a nanoindentation creep test. Petrovic et al (2005) used Mittag - Leffler function to model viscoelastic properties of dentin given by Jantarat et al (2002). The analysis provided in the mentioned work, however, is formally mathematical and does not provide any physical guidance regarding the choice of particular values of the model parameters.

To the best of our knowledge the micromechanical model for viscoelastic properties of dentin has never been proposed in literature. This process is complicated by the lack of solid information on mechanical behavior of dentin, in general, and its creep-relaxation behavior, in particular. Generally, the approach to find analytical solution of the homogenization problem for a heterogeneous material with viscoelastic constituents is based on elasticity-viscoelasticity correspondence principle. The problem is formulated in the Fourier or Laplace domain, treated as the elastic one, and then, inverse transform gives the desired viscoelastic solution. The main challenge appearing in this approach is to obtain analytical formulas for the inverse transform. It can be done only in some simplistic cases represented as combinations of dashpots and springs. Unfortunately, these models are not sufficiently flexible to match experimental data for real materials. An alternative approach has been proposed by Scott Blair and Coppen (1939 Scott Blair and Coppen (1943) (based on experimental observations) and by Rabotnov (1948) (theoretically) (hereafter –SBR model). They suggested to use fraction-exponential operators that, on one hand can describe experimental data of real materials with sufficient accuracy and, on the other hand, allow the inverse Laplace transforms in an explicit analytical form. Detailed description of the approach is given, for example, in the books of Rabotnov (1977) and Gorenflo et al. (2014). Methodology of application of fraction-exponential operators to heterogeneous materials has been recently developed by Sevostianov and Levin (2016) who introduced creep and relaxation contribution tensors that allow description of the effect of inhomogeneities on the overall viscoelastic properties in a unified way and thus, extend any of known micromechanical schemes from elastic materials to viscoelastic ones. The approach has been successfully verified by Sevostianov et al. (2016) to calculate effective viscoelastic properties of fiber reinforced composites.

The present paper constitutes the first step toward development of a micromechanical model of viscoelastic properties of dentin. We performed compressive quasi-static creep tests on crown dentin specimens (cut from molars, Figure 1, left) and reconstructed the four physical parameters of the SBR viscoelastic model – instantaneous and long-time elastic moduli, relaxation time and its power. We also applied the approach to experimental data of Jantarat et al (2002) for root dentin (cut from incisors, Figure 1, right) and compared viscoelastic parameters for two different sets of data.

## 2. Experimental measurement of viscoelastic properties of dentin

Three intact human molars were used in this work. They did not contain damages and were extracted from mature subjects (25-40 years old) according to the medical diagnosis and the Ethic Protocol of the Urals State Medical University at Yekaterinburg, Russia. Detailed description of the methods of sample preparation was given Zaitsev et al (2012). Five samples were cut from the crown of teeth by means of a diamond saw with water irrigation and, further, their surfaces were abraded using the abrasive papers for removing the damaged layer on the back surfaces of the samples. The samples had parallelepiped shape with dimensions $2 \times 2 \times 0.7$ mm$^3$ (see Figure 2).

Uniaxial compression was carried out by means of a Shimadzu AGX-50 kN (Japan) testing machine at room temperature under creep conditions (constant stress). The rate of initial loading was 0.1 mm/min for all tests. The held stresses, 300MPa and 450MPa, were applied to the samples during 5 hours, and the relaxation of deformation was controlled after unloading of the samples in during 12 minutes. Processing of the results was carried out by Trapezium-X software. The lateral deformation of the sample was calculated using a Canon photo microsystem (Japan), where the width of the sample was compared in situ with the etalon (Cu wire $L$ = 3.36 mm). The axial deformation of the sample was measured by the testing machine.

## 3. Elastic-viscoelastic analogy in terms of fraction-exponential operators

To describe the viscoelastic behavior of dentin, we use the most general form of the governing equation in the form

$$\varepsilon_{ij}(x,t)=S_{ijkl}\sigma_{kl}(x,t)+\int_0^t K_{ijkl}(t-\tau)\sigma_{kl}(x,\tau)d\tau \tag{3.1}$$

where $\varepsilon_{ij}$ and $\sigma_{kl}$ are the strain and the stress tensors, respectively, $S_{ijkl}$ is a fourth rank tensor of instantaneous elastic compliance and $K_{ijkl}(t)$ is time dependent forth rank tensor (the creep kernel) satisfying the fading memory principle, i.e. $K_{ijkl}(t)\xrightarrow[t\to\infty]{}0$. The most widely used approach to solve boundary value problems for linear viscoelastic materials is based on Laplace (or other integral) transform (see, for example, Christensen, 1982):

$$\overline{f}(p)=\int_0^\infty f(t)\mathrm{e}^{-p\mathrm{t}}dt. \quad (3.2)$$

Then, relation (3.1) may be rewritten as

$$\overline{\varepsilon}_{ij}(p)=\overline{S}_{ijkl}(p)\overline{\sigma}_{kl}(p) \quad (3.3)$$

and thus a solution for viscoelastic problem can be obtained from the corresponding elastic solution by using inverse Laplace transform. The main challenge of this approach is that only the simplest kernels in (3.1) (for example, exponential ones) allow explicit analytical inversion. In other cases, solutions for viscoelastic problems can be obtained only numerically. For most materials, however, the simplest exponential kernels do not fit experimental data properly.

Scott Blair and Coppen (1939 Scott Blair and Coppen (1943) and Rabotnov (1948) proposed to use fraction-exponential function

$$\grave{E}_\alpha(\beta,t-\tau)=(t-\tau)^\alpha\sum_{n=0}^{\infty}\frac{\beta^n(t-\tau)^{n(1+\alpha)}}{\Gamma[(n+1)(1+\alpha)]},(-1<\alpha<0) \quad (3.4)$$

for kernels in viscoelastic operators that allow explicit analytical solution using Laplace transform and, at the same time, are sufficiently general to provide a good agreement with the experimental data. To satisfy the fading memory principle, the following restrictions on the parameters entering (3.4) have to be satisfied:

$$\beta<0;\ -1<\alpha\leq 0 \quad (3.5)$$

Operator with such a kernel acts onto constant function $c$ as follows:

$$\grave{E}_\alpha(\beta)\cdot c=\frac{c}{\beta}\left[\mathrm{M}_{1+\alpha}\left(\beta t^{1+\alpha}\right)-1\right] \quad (3.6)$$

where $M_\lambda(z)$ is the Mittag-Lefler's function (Gorenflo *et al,* 2014):

$$M_\lambda(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(n\lambda+1)} \quad (3.7)$$

which decreases monotonically from 1 to 0 so that

$$\lim_{t\to\infty}\left[\grave{E}_\alpha^{*}(\beta,t)\cdot 1\right]=\frac{-1}{\beta} \quad (3.8)$$

For uniaxial tension or compression (relevant to the present study), expression (3.1) takes the following form (Christensen, 1982)

$$\varepsilon(x,t)=(E^{*})^{-1}\sigma \quad (3.9)$$

where operator of creep $(e^{*})^{-1}$ is inverse to the operator of relaxation $E^{*}$. These operators can be written in terms of Scott Blair –Rabotnov (SBR) kernel (3.4) as

$$\begin{aligned}(E^{*})[\varepsilon(x,t)]&=E_0\left[\varepsilon(x,t)+\lambda\int_0^t\grave{E}_\alpha(\beta,t-t')\varepsilon(x,t')dt'\right]\\(E^{*})^{-1}[\sigma(x,t)]&=\frac{1}{E_0}\left[\sigma(x,t)-\lambda\int_0^t\grave{E}_\alpha(\beta-\lambda,t-t')\sigma(x,t')dt'\right]\end{aligned} \quad (3.10)$$

where $E_0$ is the instantaneous Young's modulus in the direction of loading. This formula, in particular, clarifies the physical meaning of parameter $\beta$ - it is the inverse of the relaxation time $\tau$ to the power of $1+\alpha$ taken with the negative sign:

$$\beta=-\frac{1}{\tau^{1+\alpha}} \quad (3.11)$$

From (3.10) and (3.11) it follows that

$$\lambda=\beta(1-\varepsilon_{\max})=\frac{E_0-E_\infty}{E_0}\beta \quad (3.12)$$

where $E_\infty$, is Young modulus at $t \to \infty$ and $\varepsilon_{max}$ is the maximal strain in the direction of loading. Therefore, the uniaxial viscoelastic behavior of a material is described by four parameters: $E_0$, $\alpha$, $\beta$ (or $\tau$), and $\lambda$ (or $E_\infty$). Since in the processes of creep and relaxation the Young's modulus is a decreasing function of time ($E_0 \geq E_\infty$), we have

$$\beta<\lambda<0. \quad (3.13)$$

For $\alpha = 0$, the kernel of SBR operator is reduced to the ordinary exponential function. In this case it describes the properties of standard viscoelastic material (Kelvin material) representing combination of two springs with stiffnesses $E_1$ and $E_2$, and a dashpot of viscosity $\eta$:

$$\left(\frac{1}{E_1}+\frac{1}{E_2}\right)\sigma+\frac{\eta}{E_1E_2}\dot{\sigma}=\varepsilon+\frac{\eta}{E_2}\dot{\varepsilon} \quad (3.14)$$

The convenience of SBR operator is that the algebra of these operators is well developed (see Rabotnov, 1977). In particular, we have

$$\begin{gathered} \grave{E}^*_\alpha(\beta_1)\cdot\grave{E}^*_\alpha(\beta_2)=\frac{\grave{E}^*_\alpha(\beta_1)-\grave{E}^*_\alpha(\beta_2)}{\beta_1-\beta_2}, \quad (\beta_1\neq\beta_2) \\ \grave{E}^*_\alpha(\beta)\cdot\grave{E}^*_\alpha(\beta)=\frac{\partial}{\partial\beta}\grave{E}^*_\alpha(\beta) \end{gathered} \quad (3.15)$$

Laplace transform of the kernel has the following form:

$$\mathrm{L}\left[\grave{E}^*_\alpha(\beta,t)\right]\equiv\int_0^\infty\grave{E}^*_\alpha(\beta,t)e^{-tp}dt=\frac{1}{p^{1-\alpha}+\beta} \quad (3.16)$$

Therefore, if the elastic solution can be represented as a rational function of parameter $x = p^{1+\alpha}$, then its inverse Laplace transform can be obtained analytically in explicit form. In the text to follow we represent the experimental data on creep of dentin in terms of the SBR operators.

## Remark

In the last decade, fraction exponential operators have been widely used in various applications (see Podlubny, 1998). Sometimes it is done just formally without accounting for physical meaning of the model parameters. In particular, description of viscoelastic properties of dentin in terms of Mittag-Lefler function given by Petrovic et al (2005) does not include discussion of their parameters (17) and thus complicates practical applications of the model.

## 4. Reconstruction of the fraction-exponential parameters from experimental data

A custom Matlab script was developed to determine the viscoelastic parameters $\alpha, \beta$ and $\lambda$ from experimental data using nonlinear least squares fitting. Trust Region Reflective method was used to minimize the squared second norm of the vector **e** denoting the error between the SBR analytical model and experimental data:

$$\min_{(\alpha,\beta,\lambda)}\|\mathbf{e}\|_2^2 = \min_{(\alpha,\beta,\lambda)}\left\|\frac{\varepsilon(\sigma^{(0)},\alpha,\beta,\lambda,\mathbf{t}) - \varepsilon_{\exp}(\sigma^{(0)},\mathbf{t})}{\bar{\varepsilon}_{\exp}(\sigma^{(0)})}\right\|_2^2, \quad (4.1)$$

where $\boldsymbol{\varepsilon}(\sigma^{(0)}, \alpha, \beta, \eta)$ is the vector of strain values calculated using SBR model for the given set of viscoelasticity parameters, applied loading $\sigma^{(0)}$ and time instants $\mathbf{t}$, $\boldsymbol{\varepsilon}_{exp}(\sigma^{(0)}, \mathbf{t})$ is the vector of experimentally obtained strain values for loading $\sigma^{(0)}$ and time instants $\mathbf{t}$, and $\bar{\varepsilon}_{exp}(\sigma^{(0)})$ is the mean value of strain $\boldsymbol{\varepsilon}_{exp}(\sigma^{(0)}, \mathbf{t})$. The normalization by $\bar{\varepsilon}_{exp}(\sigma^{(0)})$ is used when datasets from the same material obtained under different loading are fitted simultaneously. In such cases,additional constraints on the minimization procedure need to be imposed. While in theory all thethree parameters, $\alpha$, $\beta$ and $\lambda$ of a material may depend on the applied stress, we use the same values of $\alpha$ for different loading levels. Moreover, the long-time modulus $E_\infty$ at $t \to \infty$ must be the same regardless of the loading. Given two datasets for applied stresses $\sigma_1^{(0)}$ and $\sigma_2^{(0)}$, the latter condition puts a constraint on the pair of sets of coefficients $(\beta_1, \lambda_1)$ and $(\beta_2, \lambda_2)$, and can be implemented by introducing an additional component to the vector **e**:

$$e_{N+1} = \frac{\lambda_1}{\beta_1} - \frac{\lambda_2}{\beta_2}, \quad (4.2)$$

where $N$ is the combined number of points in two datasets.

The instantaneous modulus $E_0$ of a given material was calculated as an average of slopes of stress/strain curves for different values of $\boldsymbol{\sigma}^{(0)}$ during initial loading. All points pertaining to loading and unloading stages were excluded from the least squares fitting analysis used to determine the parameters $\alpha$, $\beta$ and $\lambda$.

### Viscoelastic parameters of crown dentin of a molar

Procedure described above was used to determine the viscoelastic parameters of crown dentin, see Section 2 for details on specimen preparation and testing procedures. The instantaneous ($E_0$) and long-time ($E_\infty$) moduli were found to be 5.13 *GPa* and 1.83 *GPa* correspondingly. Experimental data for $\sigma_1^{(0)} = 300\ \mathrm{MPa}$ and $\sigma_2^{(0)} = 450\ \mathrm{MPa}$ were fitted and the following parameters were obtained:

$$\alpha_1 = -0.8231,\ \beta_1 = -1.5076,\ \lambda_1 = -0.9705$$
$$\alpha_2 = -0.8231,\ \beta_2 = -1.0692,\ \lambda_2 = -0.6884$$

The experimental data and the resulting analytical models are shown in Figure 3.

### Viscoelastic parameters of root dentin of an incisor

Creep test results reported by Jantarat et al (2002) were processed to determine the SBR model parameters of root dentin. The instantaneous ($E_0$) and long-time ($E_\infty$) moduli were found to be 12.18 *GPa* and 4.10 *GPa* correspondingly. Experimental data for $\sigma_1^{(0)} = 12.73\,\mathrm{MPa},\ \sigma_2^{(0)} = 38.20\mathrm{MPa},\ \sigma_3^{(0)} = 63.66\mathrm{MPa}$ and $\sigma_4^{(0)} = 89.13\mathrm{MPa}$ were fitted and the following parameters were obtained:

$$\alpha_1 = -0.9131,\ \beta_1 = -1.1260,\ \lambda_1 = -0.7473$$
$$\alpha_2 = -0.9131,\ \beta_2 = -0.3810,\ \lambda_2 = -0.2528$$
$$\alpha_3 = -0.9131,\ \beta_3 = -0.2159,\ \lambda_3 = -0.1433$$
$$\alpha_4 = -0.9131,\ \beta_4 = -0.1709,\ \lambda_4 = -0.1134$$

The experimental data and the resulting analytical models are shown in Figure 4.

## 5. Discussion and conclusions

In the process of the extraction of viscoelastic properties of dentin discussed in Section 4, we have assumed that the instantaneous and long-time Young's moduli are properties of a material and may depend on volume fractions of its constituents, collagen and hydroxyapatite, but not on the level of stresses. Thus the ratio $\lambda/\beta$ is to be stress independent as well. On the other hand, parameter $\beta$ is inversely proportional to the relaxation time to positive power $(1 + \alpha)$ and, therefore has to be a decreasing function of applied stress. We have made a conservative assumption about the stress-independency of $\alpha$, but this assumption may be an unnecessary restriction.

To construct the bounds for the instantaneous Young's modulus $E_0$ we represented dentin as a composite consisting of the collagen fibers and hydroxyapatite crystals. Young moduli of hydroxyapatite and collagen are taken as $E_0^{\mathrm{HA}} = 114.0\mathrm{GPa}$ (Gilmore and Katz, 1982) and $E_0^C = 1.5\mathrm{GPa}$ (Currey, 1969). Poisson's ratio for both materials is taken as $\nu = 0.3$, in accordance with the available data on the shear moduli for these materials (Gilmore and Katz, 1982; Tanioka *et al*, 1974).

Voigt-Reuss bounds for bulk, $K$, and shear, $G$, moduli of a two phase composite (phases 1 and 2 are marked by corresponding superscripts) with volume fractions of the constituents $c_1$ and $c_2 = 1 - c_1$ are given by (Hill, 1963)

$$K_R \le K \le K_V,\ G_R \le G \le G_V \quad (5.1)$$

where

$$K_V = c_1K_1 + c_2K_2\ \frac{1}{K_R} = \frac{c_1}{K_1} + \frac{c_2}{K_2},\ G_V = c_1G_1 + c_2G_2; \frac{1}{G_R} = \frac{c_1}{G_1} + \frac{c_2}{G_2} \quad (5.2)$$

The bounds for the Young's modulus are obtained by expressing the latter in terms of bulk and shear moduli as

$$E = 9KG/(3K+G). \quad (5.3)$$

The Voight estimate predicts a stiffer response than the Reuss one with the difference being of the second order in elastic contrast:

$$K_V - K_R = \frac{(K_1 - K_2)^2}{K_1/c_1 + K_2/c_2}\ \mu_V - \mu_R = \frac{(\mu_1 - \mu_2)^2}{\mu_1/c_1 + \mu_2/c_2} \quad (5.4)$$

For high contrast between elastic properties of two phases (like in the case of hydroxyapatite and collagen), it yields rather wide bounds. For an isotropic statistically homogeneous material, one can use HashinShtrikman bounds (Hashin and Shtrikman, 1963):

$$K_{\mathrm{HS}}^{-} \le K \le K_{\mathrm{HS}}^{+},\ G_{\mathrm{HS}}^{-} \le G \le G_{\mathrm{HS}}^{+} (5.5) \quad (5.5)$$

where

$$\begin{aligned} K_{\mathrm{HS}}^{-} &= K_1 + \frac{c_2}{\frac{1}{K_2 - K_1} + \frac{3c_1}{3K_1 + 4G_1}}; K_{\mathrm{HS}}^{+} = K_2 + \frac{c_1}{\frac{1}{K_1 - K_2} + \frac{3c_2}{3K_2 + 4G_2}} \\ G_{\mathrm{HS}}^{-} &= G_1 + \frac{c_2}{\frac{1}{G_2 - G_1} + \frac{6c_1(K_1 + 2G_1)}{5G_1(3K_1 + 4G_1)}}; G_{\mathrm{HS}}^{+} = G_2 + \frac{c_1}{\frac{1}{G_1 - G_2} + \frac{6c_2(K_2 + 2G_2)}{5G_2(3K_2 + 4G_2)}} \end{aligned} \quad (5.6)$$

Hashin-Shtrikman and Voigt-Reuss bounds for the collagen-hydroxyapatite composite are presented in Figure 5. The dashed area corresponds to the possible values of the hydroxyapatite volume fraction in dentin. The extracted values of $E_0$ for both sets of the data are inside the bounds.

As expected, the long-time Young's modulus ($E_\infty$) is smaller than the instantaneous one ($E_0$). For both sets of the experimental data, the ratio between these two values is about 3. With increasing level of stress, the values of parameter $\beta$ decrease (that corresponds to increasing relaxation time). Thus the values of all the extracted parameters of the fraction-exponential model are in the range of the expected intervals of variation corresponding to their physiiscoelastic behavior of dentin yields a relatively simple method cal meaning. The proposed method of the description of viscoelastic behavior of dentin yields a relatively simple method for solution of mechanical problems for this material (using Laplace transform) and provides a good accuracy as seen in Figures 3 and 4.

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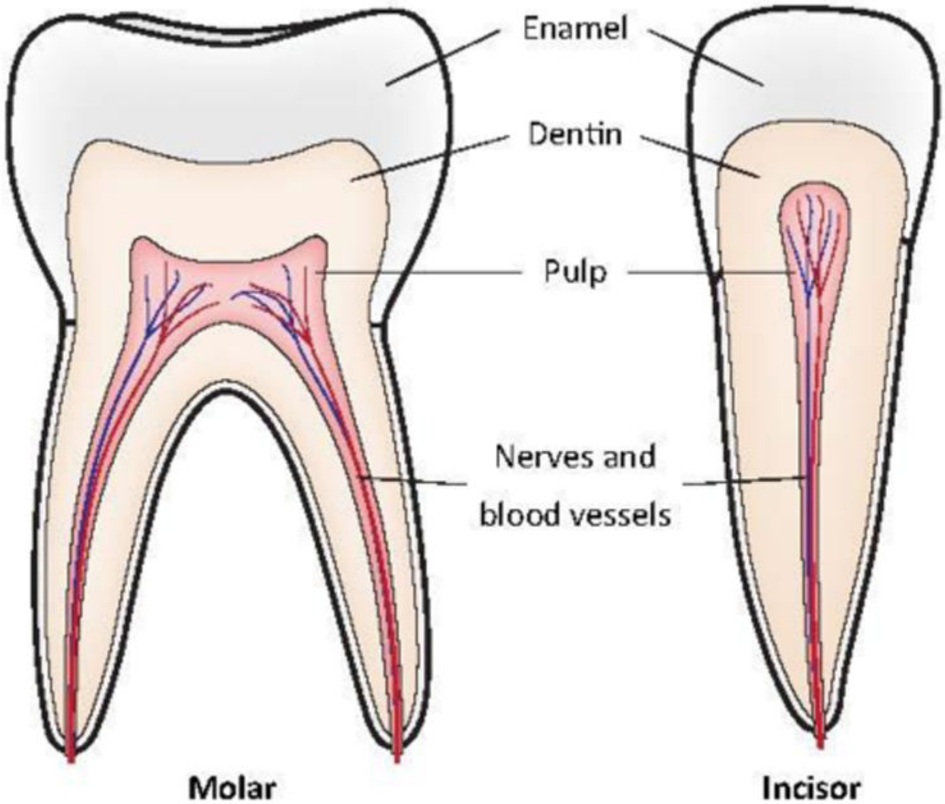


**Figure 1.**
Microstructure of molar and incisor.

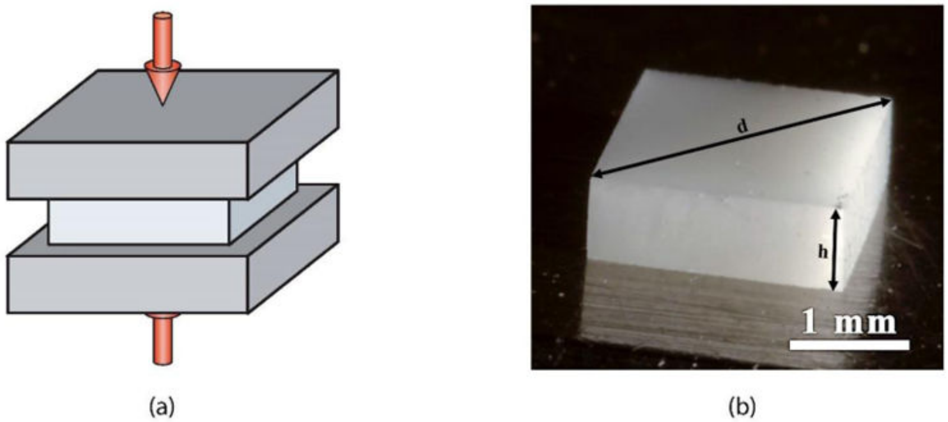


**Figure 2.**
Experimental setup of creep under compression test.

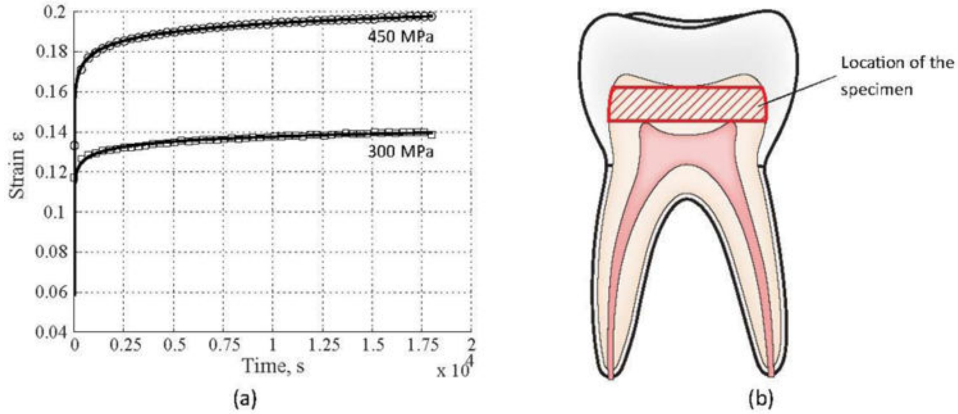


**Figure 3.**

(a) Creep test results (data points) and SBR model (solid lines) for crown dentin for two stress levels; (b) location of the specimens within a molar tooth

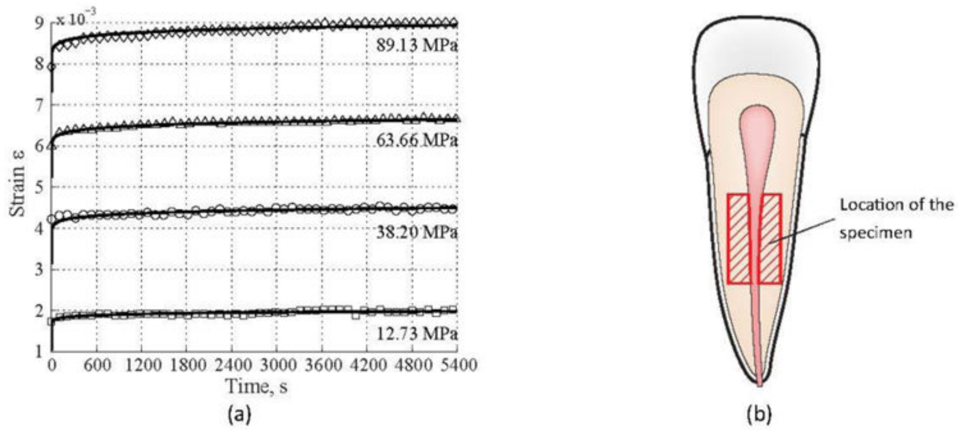


**Figure 4.**

(a) Creep test results of Jantarat et al (2002) (data points) and SBR models (solid lines) for root dentin for four stress levels; (b) location of the specimens within an incisor tooth

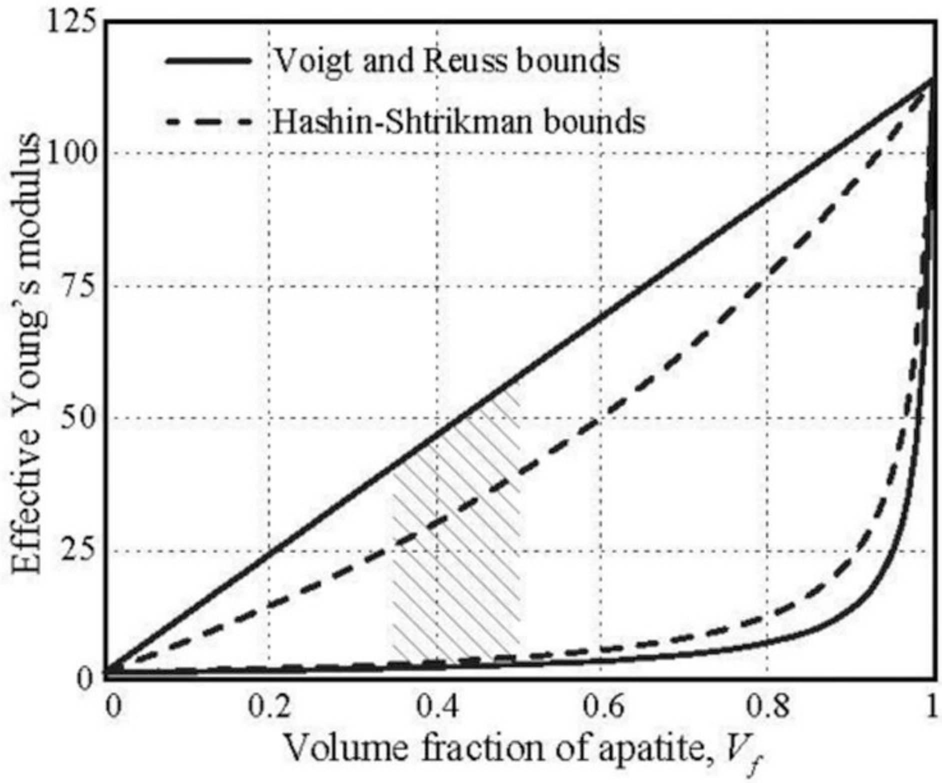


**Figure 5.**
Voigt-Reuss and Hashin-Shtrikman bounds for collagen-hydroxyapatite composite as functions of the volume fraction of hydroxy apatite. The shaded region corresponds to the interval of possible variation of the hydroxyapatite content in dentin.